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PARTON EQUILIBRATION IN ULTRARELATIVISTIC HEAVY ION COLLISIONS[1]

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ABSTRACT

Medium effects which include color screening and Landau-Pomeranchuk-Migdal suppression of induced radiation are discussed in connection with the equilibration of dense partonic system in ultrarelativistic heavy ion collisions. Taking into account of these medium effects, the equilibration rate for a gluonic gas are derived and the consequences are discussed.

1. Introduction

Strong interactions involved in hadronic collisions can be generally divided into two categories depending on the scale of momentum transfer Q^2 of the processes. When $Q^2 \sim \Lambda^2_{QCD}$, the collisions are nonperturbative in QCD and are considered soft. These interactions can be approximated by some effective theories in which partons inside a nucleon cannot be resolved and nucleons interact coherently by the exchange of mesons or soft Pomerons. These kind of coherent interactions will result in the collective excitations as observed in experiments at low and intermediate energies, $\sqrt{s} <$ a few GeV. On the other hand, if Q^2 is much larger than Λ^2_{QCD}, parton model becomes relevant and they interact approximately incoherently. Due to the high Q^2, parton interactions can be calculated via perturbative QCD (pQCD), given the initial parton distribution functions inside a nucleus. At high collider energies as RHIC and LHC, the hard processes of parton interactions become dominant and heavy ion collisions are generally very violent[1]. In these violent collisions, it is quite likely that colors are liberated during the collisions and we can start with a color deconfined system.

In the last few years, many efforts have been made to estimate the initial parton density by perturbative calculations[2,3,4,5]. Though the results depend on parton structure functions inside a nucleus and the modeling of the underlying soft interactions,

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a general consensus is that a dense partonic system will be produced during the early stage of ultrarelativistic heavy ion collisions. However, the system is not necessarily in both thermal and chemical equilibrium. The initial system is totally dominated by gluons and the quark and anti-quark densities are far below their chemical equilibrium values. Numerical simulations of parton cascade can be made with certain simplifications. However, the complexity of these calculations makes it difficult to obtain a clear understanding of the different time scales on various parameters and model assumptions. Most importantly, medium effects and interferences are difficult to incorporate. In this talk, I would like to describe two medium effects. Color screening is caused by the interaction of a color charge with the medium which gives a natural infrared cut-off to regularize the parton cross sections inside the dense system[6]. Another medium effect is the so-called Landau-Pomeranchuk-Migdal effect of the reduced radiation by multiple scattering through the medium[7]. These medium effects can be utilized to obtain a parameter-free set of equations, based on perturbative QCD in a dense partonic medium, that describes the evolution of quark and gluon distributions towards equilibrium[8].

2. Color Screening

Let us first look at the color screening in the initially produced parton system. Following the standard calculation(in Coulomb gauge) of screening in the time-like gluon propagator in a medium of gluonic excitation, we have the screening mass[6],

$$\mu_D^2 = -\frac{3\alpha_s}{\pi^2} \lim_{|\mathbf{q}|\to 0} \int d^3k \frac{|\mathbf{k}|}{\mathbf{q}\cdot\mathbf{k}} \mathbf{q}\cdot\nabla_{\mathbf{k}} f(\mathbf{k}). \tag{1}$$

Now, instead of using the Bose-Einstein distribution for the thermalized case, we simply relate the phase space density $f(\mathbf{k})$ to the initial gluon distribution calculated from pQCD. At high energies we find that the transverse and longitudinal screening lengths are very close. We assume that the gluon momentum distribution is similar to a thermal distribution

$$f_g(k) = \lambda_g e^{-u\cdot k/T}, \tag{2}$$

with λ_g and T characterizing the thermalization of the gluon gas. With this distribution, one obtain the effective color screening mass

$$\mu_D^2 = \lambda_g g^2 T^2. \tag{3}$$

We will use this effective color screening mass as an infrared cut-off for parton interaction cross sections inside a non-equilibrium gluonic gas.

3. Induced Radiation

The leading processes which contribute to the gluon equilibration are those with induced gluon radiation. Let us consider the simplest case of induced radiation from a

two-quark scattering. The Born amplitude for two-quark scattering $(p_i, k_i) \rightarrow (p_f, k_f)$ through one gluon exchange is,

$$\mathcal{M}_{el} = ig^2 T^a_{AA'} T^a_{BB'} \frac{\bar{u}(p_f)\gamma_\mu u(p_i)\bar{u}(k_f)\gamma^\mu u(k_i)}{(k_i - k_f)^2}, \tag{4}$$

where A, A', B, and B' are the initial and final color indices of the beam and target partons. The corresponding elastic cross section is,

$$\frac{d\sigma_{el}}{dt} = C^{(1)}_{el} \frac{\pi\alpha_s^2}{s^2} 2 \frac{s^2 + u^2}{t^2}, \tag{5}$$

where $C^{(1)}_{el} = C_F/2N = 2/9$ is the color factor for a single elastic quark-quark scattering and s,u, and t are the Mandelstam variables.

Taking into account the dominant contribution, the radiation amplitude from a single scattering is,

$$\begin{aligned} \mathcal{M}_{rad} &\equiv i\frac{\mathcal{M}_{el}}{T^a_{AA'}T^a_{BB'}}\mathcal{R}_1, \\ \mathcal{R}_1 &\simeq 2ig\vec{\epsilon}_\perp \cdot \left[\frac{\mathbf{k}_\perp}{k_\perp^2} + \frac{\mathbf{q}_\perp - \mathbf{k}_\perp}{(\mathbf{q}_\perp - \mathbf{k}_\perp)^2}\right] T^a_{AA'}[T^a, T^b]_{BB'}, \end{aligned} \tag{6}$$

where $\mathcal{M}_{el}$ is the elastic amplitude as given in Eq. (4), and $\mathcal{R}_1$ is defined as the radiation amplitude induced by a single scattering. For later convenience, all the color matrices are included in the definition of the radiation amplitude $\mathcal{R}_1$. With the above approximations, we then recover the differential cross section for induced gluon bremsstrahlung by a single collision as originally derived by Gunion and Bertsch [9],

$$\frac{d\sigma}{dtdyd^2k_\perp} = \frac{d\sigma_{el}}{dt}\frac{dn^{(1)}}{dyd^2k_\perp}, \tag{7}$$

where the spectrum for the radiated gluon is,

$$\frac{dn^{(1)}}{dyd^2k_\perp} \equiv \frac{1}{2(2\pi)^3 C^{(1)}_{el}}\overline{|\mathcal{R}_1|^2} = \frac{C_A\alpha_s}{\pi^2}\frac{q_\perp^2}{k_\perp^2(\mathbf{q}_\perp - \mathbf{k}_\perp)^2}. \tag{8}$$

In the square modulus of the radiation amplitude, an average and a sum over initial and final color indices and polarization are understood. The above formula is also approximately valid for induced radiation off a gluon line, except that the color factor $C^{(1)}_{el}$ in the elastic cross section has to be replaced by 1/2 for gq and 9/8 for gg scatterings.

One nonabelian feature in the induced gluon radiation amplitude, Eq. (6), is the singularity at $\mathbf{k}_\perp = \mathbf{q}_\perp$ due to induced radiation along the direction of the exchanged gluon. For $k_\perp \ll q_\perp$, we note that the induced radiation from a three gluon vertex can be neglected as compared to the leading contribution $1/k_\perp^2$. However, at large

$k_\perp \gg q_\perp$, this three gluon amplitude is important to change the gluon spectrum to a $1/k_\perp^4$ behavior, leading to a finite average transverse momentum. Therefore, $q_\perp$ may serve as a cut-off for $k_\perp$ when one neglects the amplitude with the three gluon vertices as we will do when we consider induced radiation by multiple scatterings in the next section.

4. EFFECTIVE FORMATION TIME

The radiation amplitude induced by multiple scatterings has been discussed in Ref. [7]. We here only briefly discuss the case associated with double scatterings. We consider two static potentials separated by a distance L which is much larger than the interaction length, $1/\mu$. For convenience of discussion we neglect the color indices in the case of an abelian interaction first. The radiation amplitude associated with double scatterings is,

$$\mathcal{R}_2^{\rm QED} = ie\left[\left(\frac{\epsilon\cdot p_i}{k\cdot p_i} - \frac{\epsilon\cdot p}{k\cdot p}\right)e^{ik\cdot x_1} + \left(\frac{\epsilon\cdot p}{k\cdot p} - \frac{\epsilon\cdot p_f}{k\cdot p_f}\right)e^{ik\cdot x_2}\right], \tag{9}$$

where $p = (p_f^0, p_z, \mathbf{p}_\perp)$ is the four-momentum of the intermediate parton line which is put on mass shell by the pole in one of the parton propagators, $x_1 = (0, \mathbf{x}_1)$, and $x_2 = (t_2, \mathbf{x}_2)$ are the four-coordinates of the two potentials with $t_2 = (z_2 - z_1)/v = Lp^0/p_z$. We notice that the amplitude has two distinguished contributions from each scattering. Especially, the diagram with a gluon radiated from the intermediate line between the two scatterings contributes both as the final state radiation for the first scattering and the initial state radiation for the second scattering. The relative phase factor $k\cdot(x_2 - x_1) = \omega(1/v - \cos\theta)L$ then will determine the interference between radiations from the two scatterings. If we define the formation time as

$$\tau(k) = \frac{1}{\omega(1/v - \cos\theta)} \simeq \frac{2\omega}{k_\perp^2}, \tag{10}$$

then Bethe-Heitler limit is reached when $L \gg \tau(k)$. In this limit, the intensity of induced radiation is simply additive in the number of scatterings. However, when $L \ll \tau(k)$, the final state radiation from the first scattering completely cancels the initial state radiation from the second scattering. The radiation pattern looks as if the parton has only suffered a single scattering. This is often referred to as the Landau-Pomeranchuk-Migdal (LPM) effect[10]. The corresponding limit is usually called factorization limit.

The extrapolation to the general case of m number of scatterings gives us the radiation amplitude in QCD,

$$\mathcal{R}_m = i2g\frac{\vec{\epsilon}\cdot\mathbf{k}_\perp}{k_\perp^2}T^{a_1}_{A_1A_1'}\cdots T^{a_m}_{A_mA_m'}\sum_{i=1}^{m}\left(T^{a_m}\cdots[T^{a_i},T^b]\cdots T^{a_1}\right)_{BB'}e^{ik\cdot x_i}, \tag{11}$$

which contains m terms each having a common momentum dependence in the high energy limit, but with different color and phase factors. The above expression should also be valid for a gluon beam jet, with the corresponding color matrices replaced by those of an adjoint representation. The spectrum of soft bremsstrahlung associated with multiple scatterings in a color neutral ensemble is, similar to Eq. (8),

$$\frac{dn^{(m)}}{dyd^2k_\perp} = \frac{1}{2(2\pi)^2 C_{el}^{(m)}} \overline{|\mathcal{R}_m|^2} \equiv C_m(k) \frac{dn^{(1)}}{dyd^2k_\perp}, \tag{12}$$

where $C_{el}^{(m)} = (C_F/2N)^m$ is the color factor for the elastic scattering cross section without radiation. $C_m(k)$, defined as the "radiation formation factor" to characterize the interference pattern due to multiple scatterings, can be expressed as,

$$C_m(k) = \frac{1}{C_F^m C_A N} \sum_{i=1}^{m} \left[C_{ii} + 2Re \sum_{j=1}^{i-1} C_{ij} e^{ik\cdot(x_i - x_j)} \right], \tag{13}$$

where the color coefficients are defined as

$$C_{ij} = Tr \left(T^{a_m} \cdots [T^b, T^{a_i}] \cdots T^{a_1} T^{a_1} \cdots [T^{a_j}, T^b] \cdots T^{a_m} \right). \tag{14}$$

If we average over the interaction points $\mathbf{x}_i$ according to a linear kinetic theory, we find that an effective formation time in QCD can be defined as,

$$\tau_{\mathrm{QCD}}(k) = r_2 \tau(k) = \frac{C_A}{2C_2} \frac{2\cosh y}{k_\perp}, \tag{15}$$

which depends on the color representation of the jet parton. The induced gluon radiation due to multiple scattering will be suppressed when the mean free path λ is much smaller than the effective formation time.

5. Equilibration Rates

The most important reactions for establishing gluon equilibrium are $gg \leftrightarrow ggg$. Elastic scattering processes, on the other hand, are crucial for maintaining local thermal equilibrium. Multi-gluon radiation is presumably suppressed by color screening, while radiative processes involving quarks have smaller cross sections due to QCD color factors.

The evolution of the gluon density n according to reactions mentioned above can be described by a rate equation. Adding the equation for energy conservation assuming only longitudinal expansion we end up with a closed set of equations determining the temperature $T(\tau)$ and the gluon "fugacity" $\lambda_g(\tau) \equiv n/n_{eq}(T)$ as a function of the proper time τ,

$$\frac{\dot{\lambda}_g}{\lambda_g} + 3\frac{\dot{T}}{T} + \frac{1}{\tau} = R_3(1 - \lambda_g), \tag{16}$$

$$\lambda_g^{3/4} T^3 \tau = \mathrm{const.} \tag{17}$$

This set of evolution equations is completely controlled by the gluon production rate, $R_3 = \frac{1}{2}\sigma_3 n$.

Taking into account of LPM effect, the cross section for $gg \to ggg$ processes can be written as,

$$\frac{d\sigma_3}{d^2q_\perp dy d^2k_\perp} = \frac{d\sigma_{el}}{d^2q_\perp}\frac{dn^{(1)}}{d^2k_\perp dy}k_\perp \cosh y\theta(\lambda - \tau_{\rm QCD}(k))\theta(E - k_\perp \cosh y), \tag{18}$$

where $\tau_{\rm QCD}(k)$ is given by Eq. (15), the second θ-function is for energy conservation. The gluon density distribution induced by a single scattering is given by Eq. (8) which must be regularized by the color screening mass μ_D in Eq. (3). Using the elastic cross section of gluon scattering regularized also by μ_D, we obtain a fugacity independent mean free path

$$\lambda^{-1} = \sigma^{2\to 2}n = \tfrac{9}{8}a_1\alpha_s T, \tag{19}$$

Using these values we evaluate the chemical gluon equilibration rate $R_3 = \frac{1}{2}n\sigma_3$, as defined in Eq. (18), numerically. This rate scales with the temperature linearly but is a complicated function of the gluon fugacity. The result can approximated by an analytical fit,

$$R_3 = 2.1\alpha_s^2 T\left(2\lambda_g - \lambda_g^2\right)^{1/2}, \tag{20}$$

which will be used in solving the time dependent rate equations. The rate we thus obtained has a nonlinear dependence on the fugacity due to the inclusion of the LPM effect and the effective color screening mass. If LMP effect were not included, the rate whould be much larger, thus would lead to a fast equilibration.

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